

Ioffe-time distribution of quarks in the photon

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Abstract:

We have analysed the Ioffe-time distribution of quarks in virtual photons using Operator Product Expansion of the correlation function that determines the matrix element of the corresponding quark string operator. The distribution for a transversally polarised photon admits a spectral representation which can be continued to the on-shell region $p^2 = 0$. The resulting model Ioffe-time distribution turns out to be larger than parametrisations of the available F_2^γ data. This result is linked to the slope of the quark distribution at the origin, which is also too large.

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Many years ago Witten [1] showed that the Operator Product Expansion (OPE) can be applied to the description of photon-photon scattering with the target photon on or near mass shell while the probing photon has a large Euclidean momentum. According to Witten's analysis the structure function of a transversal polarised photon contains a point-like part which grows logarithmically as a function of Q^2 . It arises due to the contribution of highly virtual quarks in the quark loop and can be fully predicted in perturbative QCD. The contribution of low-virtuality quarks describes the interaction of the probe with the hadronic component of the photon, which is subdominant at asymptotically large Q^2 .

It is necessary to understand this hadronic contribution to the photon structure function better because all experiments probe the photon structure at Q^2 values, for which its hadronic part is non-negligible. Because it has not yet been possible to calculate it from first principles, Vector Meson Dominance (VDM) type models are used to estimate its magnitude. To link photon properties more directly to QCD, Gorsky et al. [2] calculated the hadronic part of the photon structure function applying OPE to the photon-photon forward Compton amplitude, which resulted in an expansion in inverse powers of the target photon virtuality. This expansion admits a spectral representation which allowed an analytic continuation to the quasi real target photon regime $p^2 \sim 0$.

However, the problem with parton distributions in momentum space is that for a given value of the momentum fraction u they get contributions from both small and large longitudinal distances. To circumvent this problem the authors of [3, 4] proposed an alternative representation, which was already applied to the quark distribution of the nucleon [5]. It turned out that in complete duality to twist-2 parton distributions in momentum space, twist-2 parton distributions in coordinate space can be introduced. The coordinate space variable, called Ioffe-time, measures the longitudinal light-cone distance between the points where the hard probe was absorbed respectively emitted by the target. Derivatives of Ioffe-time distributions at the origin are given by moments of some corresponding structure functions, or equivalently [7, 8] by matrix elements of twist-2 operators of increasing dimension. From the theoretical point of view, this connection makes Ioffe-time distributions much easier to analyse than parton distributions in momentum space.

In the present paper we concentrate on quark distributions $q^\lambda(u)$ and $\bar{q}^\lambda(u)$ in a photon with momentum p and polarization λ . Following reference

[5] we introduce the Ioffe-time distribution of quarks inside the photon $Q^\lambda(z)$, which is given by the photon matrix element of the quark string operator $\hat{O}(\Delta) = \bar{\Psi}(\Delta) \not{n} [\Delta; -\Delta] \Psi(-\Delta)$,

$$\begin{aligned} 2i(p \cdot n) Q^\lambda(z) &= \langle p\lambda | \hat{O}(\Delta) | p\lambda \rangle \\ &= 2(p \cdot n) \int_0^1 du \left[q^\lambda(u) \exp(iuz) - \bar{q}^\lambda(u) \exp(-iuz) \right], \end{aligned} \quad (1)$$

where for notational convenience we have introduced two parallel light-like vectors Δ_μ and n_μ satisfying $\Delta^2 = n^2 = n \cdot \Delta = 0$. Our convention is such that for any vector a , $n \cdot a = a^+$. In equation (1) $z = 2p \cdot \Delta$ and $[\Delta; -\Delta]$ denotes the path-ordered exponential which ensures gauge invariance of the quark distribution. As we have to respect both QED and QCD [6] gauge symmetry $[\Delta; -\Delta]$ becomes the product of colour and electromagnetic parts

$$[\Delta; -\Delta] = P \exp \left[ig \int_{-1}^1 d\xi \Delta \cdot A(\xi \Delta) \right] \cdot P \exp \left[-ie \int_{-1}^1 d\eta \Delta \cdot \tilde{A}(\eta \Delta) \right], \quad (2)$$

where $A_\mu(x)$ is the gluon field and $\tilde{A}_\mu(x)$ denotes the photon field. In the following, however, we will adopt the Schwinger gauge $x_\mu \tilde{A}_\mu(x) = 0$ for the electromagnetic field, such that the second factor becomes one, and never explicitly enters the calculations.

A Taylor expansion of $\hat{O}(\Delta)$ in Δ_μ results in local operators of twist 2. The first term in this expansion is given by the traceless part of the quark energy-momentum tensor and the matrix element of its QCD part describes the longitudinal momentum fraction carried by quarks in the target. The normalization of Ioffe-time distributions is such that this matrix element is given just by the derivative of $Q^\lambda(z)$ at $z = 0$. As the photon is even under charge conjugation (1) becomes

$$\langle p\lambda | \hat{O}(\Delta) | p\lambda \rangle = 2i(p \cdot n) \int_0^1 du \left[q^\lambda(u) + \bar{q}^\lambda(u) \right] \sin(uz), \quad (3)$$

which clearly shows that the Ioffe-time z is the Fourier-conjugate to the usual Bjorken variable u .

Using the LSZ reduction formula we calculate the photon matrix element (1) as a three-point correlation between two electromagnetic currents and the operator $\hat{O}(y; \Delta)$ centered at a point y :

$$\hat{O}(y; \Delta) = \bar{\Psi}(y + \Delta) \not{n} [y + \Delta; y - \Delta] \Psi(y - \Delta). \quad (4)$$

Denoting by $M_{\mu\nu}(p, k)$ the three-point correlation function

$$M_{\mu,\nu}(p, k) = i^2 e^2 \int d^4x d^4y \exp(ipx +iky) \langle 0 | T(j_\mu(x) \hat{O}(y; \Delta) j_\nu(0)) | 0 \rangle, \quad (5)$$

we obtain

$$\langle p\lambda | \hat{O}(y; \Delta) | p\lambda \rangle = \epsilon_\mu^*(p, \lambda) M_{\mu\nu}(p, k=0) \epsilon_\nu(p, \lambda) \quad (6)$$

We remind the reader that (6) holds only in the Schwinger gauge for the photon field; in general some extra terms arise.

With the Ward identity derived in the appendix of [5] we can write $M_{\mu\nu}(p, k)$ as a sum involving two- and three point correlators:

$$\begin{aligned} M_{\mu\nu}(p, k) = & e^2 \int d^4x \exp(ip \cdot x) \int d^4y \exp(ik \cdot y) \frac{n \cdot x}{k \cdot x} \int_{-1}^1 d\xi \langle 0 | T \{ j_\mu(x) \bar{\Psi}(y + \Delta) \\ & \times [y + \Delta; y + \xi\Delta] g \gamma^\rho \Delta^\sigma G_{\rho\sigma}(y + \xi\Delta) [y + \xi\Delta; y - \Delta] \Psi(y - \Delta) j_\nu(0) \} | 0 \rangle \\ & + i e^2 \int d^4x \exp(ip \cdot x) \frac{n \cdot x}{k \cdot x} \left[\exp(ik \cdot x) \langle 0 | T(j_\mu(x; x - 2\Delta) j_\nu(0)) | 0 \rangle \right. \\ & \left. - \langle 0 | T(j_\mu(x) j_\nu(2\Delta; 0)) | 0 \rangle + (\Delta \rightarrow -\Delta) \right], \end{aligned} \quad (7)$$

where $j_\mu(x, y)$ is the point-splitted electromagnetic current operator,

$$j_\mu(x, y) = \bar{\Psi}(x) \gamma_\mu [x; y] \Psi(y). \quad (8)$$

Note that the momentum k_μ appears in the denominators in (7) and therefore it can be taken to zero only at the end of calculation after all potentially divergent terms canceled. Expression (7) is formally equivalent to (5) but allows to apply directly the techniques developed previously in [5].

In the following we distinguish between Ioffe-time distributions in transversally (T) and longitudinally (L) polarised photons, defined by

$$\begin{aligned} \frac{1}{2} \sum_{\lambda=1,2} \langle p\lambda | \hat{O}(\Delta) | p\lambda \rangle &= 2i(p \cdot n) Q^T(z) \\ \langle pL | \hat{O}(\Delta) | pL \rangle &= 2i(p \cdot n) Q^L(z) \end{aligned} \quad (9)$$

We introduce the vector

$$\tilde{p}_\mu = p_\mu - \frac{1}{2} \frac{p^2}{p \cdot n} n_\mu, \quad (10)$$

such that $\tilde{p}^2 = 0$. The polarisation tensor can then be written in terms of transverse and longitudinal parts, $d_{\mu\nu} = d_{\mu\nu}^T - d_{\mu\nu}^L$:

$$\begin{aligned} d_{\mu\nu}^T &= \sum_{\lambda=1,2} \epsilon_\mu^*(p, \lambda) \epsilon_\nu(p, \lambda) = -g_{\mu\nu} + \frac{n_\mu \tilde{p}_\nu + n_\nu \tilde{p}_\mu}{p \cdot n} \\ d_{\mu\nu}^L &= -\epsilon_\mu^*(p, L) \epsilon_\nu(p, L) = -p^2 \frac{n_\mu n_\nu}{(p \cdot n)^2}. \end{aligned} \quad (11)$$

Our main task is using OPE to calculate the three-point correlation function (5) for the case of massless quarks up to the leading non-perturbative corrections of dimension 4,

$$Q^{T,L}(z) = Q_0^{T,L}(z) + \frac{1}{(-p^2)^2} Q_4^{T,L}(z) \quad (12)$$

where $(-p^2)$ is the virtuality of the target photon and λ its polarisation.

We shall first consider the situation for transverse polarisation. The perturbative contribution $Q_0^T(z)$ to the matrix element (9) arises from the last two terms in (7). The explicit calculation [6] gives:

$$Q_0^T(z) = \frac{3\alpha}{\pi} \sum_q e_q^4 \int du \sin(u \cdot z) \left[(u^2 + \bar{u}^2) \log\left(\frac{\mu^2}{-p^2}\right) - (u^2 + \bar{u}^2) \log(u\bar{u}) - 1 \right], \quad (13)$$

where μ^2 is the ultraviolet cut-off corresponding to the normalisation point of the operator \hat{O} , and $\alpha = \frac{1}{137}$ is the electromagnetic coupling constant. Assuming a relatively low normalisation point $\mu^2 \sim \text{a few GeV}^2$, the sum over active quark flavours runs over u,d and s quarks and $\sum e_q^4 = \frac{2}{9}$. Note that (13) has been obtained using dimensional regularisation and minimal subtraction of the ultraviolet divergence. As it is well known, only the part proportional to $\log(\frac{\mu^2}{-p^2})$ does not depend on the renormalisation scheme.

Let us now consider the $Q_4^T(z)$ term in OPE (12). In general $Q_4^T(z)$ will contain terms proportional to the VEV of the dimension-4 operator $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ and terms involving dimension-4 correlators which may arise from bilocal

power corrections. BPC's are taken into account to avoid ill-defined infrared singularities. Following [9] this can be done by introducing the “effective propagator” in the form:

$$\begin{aligned}
S_{ij}^{ab}(k) &= \frac{\delta^{ab}}{3 \cdot 36} \langle \frac{\alpha_s}{\pi} G^2 \rangle (k \cdot x) \left[\Delta_{ij} \left[\log \left(\frac{s_R(-x^2)}{4} \right) - \frac{5}{3} + 2\gamma_E \right] \right. \\
&\quad \left. + \frac{1}{2} (\Delta \cdot x) \frac{\not{x}}{(-x^2)} \right] - \frac{\delta^{ab}}{3} (k \cdot x) \Delta_{ij} \left[f_R^2 m_R^2 - \frac{\alpha_s s_R^2}{480\pi^3} \right] + \dots,
\end{aligned} \tag{14}$$

where eclipses stand for terms involving higher powers of $(k \cdot x)$ and k^2 , and $m_R^2 = 1 \text{ GeV}^2$, $s_R = 1.4 \text{ GeV}^2$ are parameters found in [9]. This leads to the coefficient function $Q_4^T(z)$

$$\begin{aligned}
Q_4^T(z) &= -\frac{4\pi\alpha}{144} \langle \frac{\alpha_s}{\pi} G^2 \rangle \sum_q e_q^4 z \int_0^1 du \cos(u \cdot z) \left[5\delta(u) - \delta(\bar{u}) + 16\left[\frac{1}{u}\right]_+ - 4 \right] \\
&\quad + 4\pi\alpha \sum_q e_q^4 z \left[\frac{1}{9} \langle \frac{\alpha_s}{\pi} G^2 \rangle \left(\log \left(\frac{s_R}{-p^2} \right) + 2\gamma_E - \frac{5}{3} \right) + \frac{\alpha_s s_R^2}{120\pi^3} - 4f_R^2 m_R^2 \right],
\end{aligned} \tag{15}$$

where for any function $f(u)$

$$\int_0^1 du \left[\frac{1}{u} \right]_+ f(u) \equiv \int_0^1 du \frac{1}{u} (f(u) - f(0)). \tag{16}$$

In the case of a longitudinally polarized target photon the calculation is much less complicated because infrared divergences are absent. The dimension zero coefficient $Q_0^L(z)$ is given by

$$Q_0^L(z) = \frac{12\alpha}{\pi} \sum_q e_q^4 \int_0^1 du \sin(u \cdot z) u \bar{u}, \tag{17}$$

while for the dimension four coefficient $Q_4^L(z)$ we obtain [??]

$$Q_4^L(z) = \frac{4\pi\alpha}{18} \langle \frac{\alpha_s}{\pi} G^2 \rangle \sum_q e_q^4 z \int_0^1 du \cos(u \cdot z). \tag{18}$$

At this point it is possible to compare our exact results with that of [2], where the photon structure function $F_2(u)$ was only calculated in the region

of intermediate u . Within the logic of the present calculation $F_2(u)$ can be obtained from the expression for Ioffe-time distribution by extracting the integrand in (1) and expanding it to the first order in z . As can be easily seen our dimension zero coefficients $Q_0^T(z)$ and $Q_0^L(z)$ are consistent with the results of ref.[2]. For comparison of the dimension four coefficients we have to neglect all singular contributions to $F_2(u)$ concentrated at the boundary values $u = 0$ and $u = 1$. In general such singular contributions are crucial in calculations of moments of the structure function [9, 10]. Certainly, in the case of Ioffe-time distributions they cannot be neglected. In ref.[2] BPC's were not taken into account. However, the dimension four contribution can be reproduced if the $[\frac{1}{u}]_+$ distribution in (15) is interpreted simply as function $\frac{1}{u}$.

To continue our predictions for transversally polarised on-shell photons we construct a Mandelstam dispersion representation in $(-p^2)$:

$$Q^T(z) = A_0(z) + \int_0^\infty ds \frac{A_1(s; z)}{s - p^2} + \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{A_2(s_1, s_2; z)}{(s_1 - p^2)(s_2 - p^2)}. \quad (19)$$

In the region $s_R < -p^2 \ll \mu^2$, spectral densities $A_1(z; s)$, $A_2(s_1, s_2, z)$ can be found through discontinuities, in $p_1^2 = p^2$ and $p_2^2 = (p + k)^2$, of loop diagrams which contribute to $M_{\mu, \nu}(p, k)$, equation (5). We make the standard approximation to represent A_0 , A_1 and A_2 by a contribution from the ρ meson plus a continuum contribution with a threshold s_0 . In this way we obtain

$$\begin{aligned} A_0(z) &= C_0(z) + C_1(z) \\ A_1(s; z) &= 0 \\ A_2(s_1, s_2; z) &= \delta(s_1 - m_\rho^2) \delta(s_2 - m_\rho^2) \left(C_2(z) + \frac{1}{2} s_0^2 C_1(z) \right) \\ &\quad + \delta(s_1 - s_2) \Theta(\mu^2 - s_1) \Theta(s_1 - s_0) \sqrt{s_1 s_2} C_1(z) \\ &\quad - C_3(z) \delta(s_1 - m_\rho^2) \Theta(\mu^2 - s_2) \Theta(s_2 - s_R) \frac{1}{s_2}, \end{aligned} \quad (20)$$

with the coefficient functions

$$\begin{aligned} C_0(z) &= -\frac{3\alpha}{\pi} \sum_q e_q^4 \int_0^1 du \sin(u \cdot z) [(u^2 + \bar{u}^2) \log(u \bar{u}) + 1] \\ C_1(z) &= \frac{3\alpha}{\pi} \sum_q e_q^4 \int_0^1 du \sin(u \cdot z) (u^2 + \bar{u}^2) \end{aligned}$$

$$\begin{aligned}
C_2(z) &= -\frac{4\pi\alpha}{144} \langle \frac{\alpha_s}{\pi} G^2 \rangle \sum_q e_q^4 z \int_0^1 du \cos(u \cdot z) \left[5\delta(u) - \delta(\bar{u}) + 16\left[\frac{1}{u}\right]_+ - 4 \right] \\
&\quad + 4\pi\alpha \sum_q e_q^4 z \left[\frac{1}{9} \langle \frac{\alpha_s}{\pi} G^2 \rangle (2\gamma_E - \frac{5}{3}) + \frac{\alpha_s s_R^2}{120\pi^3} - 4f_R^2 m_R^2 \right] \\
C_3(z) &= 4\pi\alpha \langle \frac{\alpha_s}{\pi} G^2 \rangle \sum_q e_q^4 \frac{1}{9} z.
\end{aligned} \tag{21}$$

This provides a representation of $Q^T(z, -p^2)$ valid for all $-p^2$,

$$\begin{aligned}
Q^T(z, -p^2) &= \frac{C_2(z) + \frac{1}{2} s_0^2 C_1(z)}{(m_\rho^2 - p^2)^2} \\
&\quad + C_0(z) + C_1(z) + \left[\log\left(\frac{\mu^2}{s_0 - p^2}\right) + \frac{p^2}{s_0 - p^2} \right] C_1(z) \\
&\quad - \frac{1}{(-p^2)(m_\rho^2 - p^2)} \log\left(\frac{s_R - p^2}{s_R}\right) C_3(z),
\end{aligned} \tag{22}$$

which can be used to extrapolate to $p^2 = 0$. For numerical calculations we chose the normalisation scale $\mu^2 = 4 \text{ GeV}^2$, and the other parameters equal to their standard values $s_0 = 1.5 \text{ GeV}^2$, $s_R = 1.4 \text{ GeV}^2$, $m_R^2 = 1 \text{ GeV}^2$, $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.012 \text{ GeV}^4$.

Note that the representation (22) has a clear physical interpretation. The first two lines arise from diagonal ρ meson to ρ meson and continuum to continuum transitions. The contribution in the third line is due to an off-diagonal $\rho \rightarrow$ continuum transition. The $\rho \rightarrow \rho$ transition is responsible for only half of the distribution at small z , the rest coming from continuum \rightarrow continuum and, much smaller, non-diagonal transitions. For small z , in the region where the present calculation can be trusted, the model Ioffe-time distribution follows practically a straight line i.e., it is practically determined in this region by the first derivative at $z = 0$ which equals to the momentum fraction carried by quarks.

The next step is to supplement the model Ioffe-time distribution for an on-shell photon by a pointlike contribution according to Witten's analysis, see [1]. Within the present formalism it amounts to the replacement of the factorisation scale μ^2 by the virtuality of the hard photon Q^2 , and adjusting the $-p^2$ independent term $C_0(z)$ accordingly

$$C_0(z) = \frac{3\alpha}{\pi} \sum_q e_q^4 \int_0^1 du \sin(u \cdot z) u [(u^2 + \bar{u}^2) \log(\frac{1}{u^2}) + 8u\bar{u} - 2] \tag{23}$$

which reproduces exactly the imaginary part of the box graph [2]. Furthermore, for comparison with relatively low $Q^2 = 5.2 \text{ GeV}^2$ data it is appropriate to neglect the perturbative evolution and treat the resulting Ioffe-time distribution as a low-scale, non-perturbative input. As can be seen from Figure 4 the structure function of the real photon turns out to be larger in the region of small z than parametrisations [12] of the available F_2^γ data.

As a consequence the momentum fraction carried by quarks inside the transversal polarised ρ meson comes out too large as well. The Ioffe-time distribution $Q^{\rho T}(z)$ for quarks in the transversal polarised ρ meson is related to the double ρ meson pole, renormalised by the coupling constant of the $\gamma \rightarrow \rho$ transition

$$C_2(z) + \frac{1}{2} s_0^2 C_1(z) = \sum_q e_q^4 \frac{4\pi\alpha}{g_V^2} m_\rho^4 Q^{\rho T}(z), \quad (24)$$

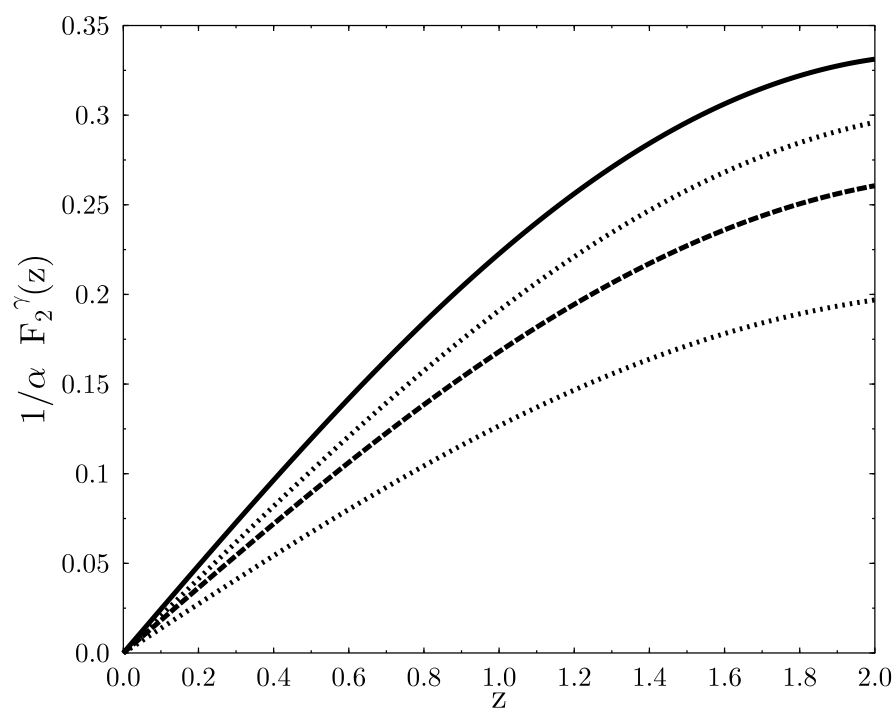
where $\frac{g_V^2}{4\pi} \sim 1.27$. The model thus predicts that essentially all longitudinal momentum is carried by quarks. Similiar problems are known from QCD sum rules calculations of the magnitude of longitudinal momentum fraction carried by quarks in a hadronic target [9, 10], although the recent calculation of the gluonic momentum fraction gave a reasonable result [11]. Moreover, due to universality of OPE, in each case the problem can be traced down to the apparently too small magnitude of negative bilocal correction (14). Clearly, we have encountered the same situation in the present context, where it can be equivalently related to a too small negative contribution from the non-diagonal transitions. When we ad hoc renormalize our Ioffe-time distribution by a factor 0.6 - 0.7, the plausible value of the momentum fraction carried by quarks in hadronic targets, the result agrees nicely with the F_2^γ data, leaving still some room for sea quark contributions. Of course such a step cannot be justified within the present formalism.

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Figure captions

Fig. 1 Real photon structure function $\frac{1}{\alpha}F_2^\gamma(z)$ (thick line) as a function of the Ioffe-time z at $Q^2 = 5.2 \text{ GeV}^2$ in comparison with a fit [12] of the available F_2^γ data (dashed line). The dotted lines indicate the uncertainty of the F_2^γ data.

Figure 1



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